10. A particle is projected with a speed of 3 ft. / sec. along the axis of x towards the origin O from an initial position at a distance 1 ft. from O on the positive side of O. After time t sec. its displacement x ft. from O is given by

$$x=Ae^{-t}+Be^{-2t},$$

where A and B are constants.

(i) Find the numerical values of A and B.

(ii) Find the time at which the particle reaches O.

(iii) Show that the particle travels beyond O, and that the time taken to travel from O to the furthest position reached beyond O is $\log_e 2$ sec.

Describe briefly the subsequent motion with the aid of a sketch graph of x plotted against t.

11. A tent covers a rectangular piece of ground of length l and breadth b. Each of the long sides of the tent is a trapezium inclined at α to the ground and each of its ends is an isosceles triangle inclined at β to the ground. Prove that

(i) the height of the tent is $\frac{1}{2}b\tan\alpha$,

(ii) the length of the top edge is $l-b\tan\alpha\cot\beta$.

Find the total surface area of the tent.

12. (*a*) Using Simpson's Rule with four intervals calculate

$$\int_0^4 e^{-x^2/50}\,dx,$$

working with four places of decimals throughout.

(*b*) A particle describes simple harmonic motion in which the displacement x is given in terms of the time t by the equation

$$x=a\sin t.$$

Find, for the interval $t=0$ to $t=\pi/2$,

(i) the mean value of the velocity with respect to time,

(ii) the mean value of the velocity with respect to distance.

UNIVERSITIES OF MANCHESTER LIVERPOOL LEEDS SHEFFIELD AND BIRMINGHAM

Joint Matriculation Board

General Certificate of Education

MATHEMATICS. PAPER I

ADVANCED

THURSDAY 17 JUNE 1954, 9.30-12.30

Answer **all** *questions in* SECTION (1) *and* **four** *questions from* SECTION (2).

SECTION (1)

1. The third term of a geometric progression is equal to the sum of the first two terms. Find, in surd form, the possible values of the common ratio.

If the first term of the progression is now taken to be 2, find, in its simplest surd form, the ' sum to infinity ' in the case when this sum is finite.

2. (*a*) Make rough sketches of the loci given by the following cartesian equations:

(i) $y^2=x$ and $y=-\sqrt{(-x)}$,

(ii) $y=x-1$ and $y=|x-1|$.

(*b*) Sketch the locus given by the polar equation $r=\theta/2\pi$, between the values $\theta=0$, $\theta=2\pi$.

3. The sides a, b and the angle A of a triangle ABC are given. State the equation giving the side c in terms of a, b and $\cos A$. Show that, if $a<b$ and $\sin A<a/b$, the difference between the two values of c is $2\sqrt{(a^2-b^2\sin^2 A)}$, and find, in terms of a, b and A, the difference in area of the two triangles which may be constructed.

 Turn over

4. Use the binomial series to write down the first four terms of the expansion of $(1+y)^{-\frac{1}{2}}$ in a series of ascending powers of y.

Hence find, in terms of $\cos\theta$, the coefficients c_1, c_2, c_3 in the expansion of

$$(1-2x\cos\theta+x^2)^{-\frac{1}{2}}$$

in the form $1+c_1x+c_2x^2+c_3x^3+\ldots$.

Prove that, when $\theta=0$, every coefficient in the series is equal to $+1$.

[You may assume throughout that the expansions are valid.]

5. (*a*) Use partial fractions to integrate

$$\frac{x}{(x+1)(x+2)}.$$

(*b*) Use the substitution $\cos\theta=x$ to find the value of

$$\int_0^{\frac{\pi}{2}} \cos^7\theta \sin^3\theta \, d\theta.$$

6. The speed v m.p.h. of a vehicle travelling from rest is given by

$$v=6x^{\frac{1}{2}}-\frac{1}{6}x^{\frac{3}{2}}$$

where x miles is the distance travelled from the start. Petrol is consumed at a rate of

$$\frac{1}{72}+\frac{5v}{1152}$$

gallons per mile when the speed is v m.p.h. Find the total amount of petrol used in travelling the first 16 miles.

Prove that in travelling this distance the maximum rate of petrol consumption is

$$\frac{1}{144}(2+5\sqrt{3})$$

gallons per mile.

SECTION (2)

Answer **four** *questions from this section.*

7. Show that the equation to the normal at the point P $(a\cos\theta,\ b\sin\theta)$ on the ellipse $x^2/a^2+y^2/b^2=1$ is

$$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2.$$

If the normal at P cuts the major and minor axes of the ellipse at G and H, show that as P moves on the ellipse the mid-point of GH describes another ellipse of the same eccentricity.

8. A point A lies on the circumference of a given circle C of radius a. With centre A an arc of a second circle S is drawn cutting C at P and Q. Find, in terms of a and θ, the area of the *sector* PAQ of S, lying within C, where the angle PAQ is 2θ radians.

Show that, when the area of the sector is a maximum, $2\theta\tan\theta=1$. Use tables to verify that this equation is satisfied when the angle PAQ is approximately 75°.

9. The equations of the sides of a triangle are

$$x+y-4=0, \quad x-y-4=0, \quad 2x+y-5=0.$$

Prove that for all numerical values of p and q the equation

$$p(x+y-4)(2x+y-5)+q(x-y-4)(2x+y-5)=(x-y-4)(x+y-4)$$

represents a curve passing through the vertices of this triangle.

Find the values of p and q which make this curve a circle, and so determine the centre and radius of the circumcircle of the triangle.